Equations

$E_n = -\frac{me^4}{8\epsilon_0^2 h^2 n^2}$ $\sigma = \frac{2\pi^5 k^4}{15h^3c^2}$

$l = mvr$ $l_x = yp_z - zp_y$

$\Delta E = h\nu = \hbar\omega$ $\lambda = h/p$

$2\pi r = n\lambda$ $\hat{P}_x = -i\hbar\frac{\partial}{\partial x}$

$\hat{H}\phi_n(x) = E_n\phi_n(x)$ $\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U(x)$

$E_n = \frac{h^2n^2}{8ma^2}$ $\Psi(x) = \sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a})$

$< a >= \int_{-\infty}^{\infty}\Psi^*(\tau)\hat{A}\Psi(\tau)d\tau$ $\sigma_a^2 =< a^2 > - < a >^2$

$\hat{H}\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$ $\Psi_n(x,t) = \psi_n(x)e^{-iE_nt/\hbar}$

$\int g^*\hat{A}f dx = \int f\hat{A}^*g^*dx$ $\psi(x) = \sum_n c_n\phi_n(x)$

$c_n = \int \phi_n^*(x)\psi(x)dx$ $p_n = |c_n|^2$

Harmonic oscillator:

$E_n = (n+\frac{1}{2})\hbar\omega$ $\omega = \sqrt{k/\mu}$

$\Psi_0(x) = (\alpha/\pi)^{1/4}\ e^{-\alpha x^2/2}$ $\Psi_1(x) = (\alpha/4\pi)^{1/4}\ 2\sqrt{\alpha}\ x\ e^{-\alpha x^2/2}$

$\alpha = \sqrt{k\mu}/\hbar$

Rigid rotor:

$E_l = \hbar^2 l(l+1)/2I$ $\Delta E = \hbar^2(l+1)/I$

$I = \mu r^2$ $\hat{H} = \hat{L}^2/2I$

Spherical harmonics:

$\hat{L}^2Y_l^m = \hbar^2 l(l+1)Y_l^m$ $l = 0,1,2,\ldots \quad -l \le m \le l$

$\hat{L}_zY_l^m = \hbar m Y_l^m$ $Y_l^m(\theta,\phi) = N_{lm}\ P_l^{|m|}(cos\theta)\ e^{im\phi}$

$Y_0^0(\theta,\phi) = 1/\sqrt{4\pi}$ $Y_1^0(\theta,\phi) = \sqrt{3/4\pi}\cos\theta$

$Y_1^1(\theta,\phi) = \sqrt{3/8\pi}\sin\theta e^{i\phi}$ $Y_1^{-1}(\theta,\phi) = \sqrt{3/8\pi}\sin\theta e^{-i\phi}$

Spin of electron:

$\hat{S}^2\alpha = \hbar^2\frac{1}{2}(\frac{1}{2}+1)\alpha$ $\hat{S}^2\beta = \hbar^2\frac{1}{2}(\frac{1}{2}+1)\beta$

$\hat{S}_z\alpha = \frac{\hbar}{2}\alpha$ $\hat{S}_z\beta = -\frac{\hbar}{2}\beta$

Some useful integrals:

$\int_{-\infty}^{\infty} e^{-ax^2}dx = \sqrt{\pi/a}$ $\int_{-\infty}^{\infty} x^2e^{-ax^2}dx = (1/2)\sqrt{\pi/a^3}$

$\int_{-\infty}^{\infty} x^4e^{-ax^2}dx = (3/4)\sqrt{\pi/a^5}$ $\int_{-\infty}^{\infty} x^6e^{-ax^2}dx = (15/8)\sqrt{\pi/a^7}$

$\int_0^a \sin^2(\frac{n\pi x}{a})dx = a/2$ $\int_0^a x\sin^2(\frac{n\pi x}{a})dx = a^2/4$

$\int_0^a x^2\sin^2(\frac{n\pi x}{a})dx = \left(\frac{a}{2\pi n}\right)^3\left(\frac{4\pi^3n^3}{3} - 2n\pi\right)$

$\int x\sin(ax)dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a}$ $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

$\int x^2\sin(ax)dx = \frac{2x}{a^2}\sin(ax) + \left(\frac{2}{a^3} - \frac{x^2}{a}\right)\cos(ax)$

Wave functions for H-like atoms: $\sigma = Zr/a_0$

$\Psi_{1s} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-\sigma}$ $\Psi_{2s} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}(2-\sigma)e^{-\sigma/2}$

$\Psi_{2p_z} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\sigma e^{-\sigma/2}\cos\theta$ $\Psi_{2p_x} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\sigma e^{-\sigma/2}\sin\theta\cos\phi$

First order perturbation correction:

$\Delta E_n = \int \psi_n^{(0)*}\hat{H}^{(1)}\psi_n^{(0)}d\tau$

Variational principle:

$E_\phi = \frac{\int \phi^*\hat{H}\phi d\tau}{\int \phi^*\phi d\tau}$ $E_\phi \ge E_0$