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We need these operators to have similar properties to the raising- and lowering operators for z

$S_x^+|+\rangle_x = 0$, $S_x^+|-\rangle_x = \hbar|-\rangle_x$, $S_x^-|+\rangle_x = \hbar|-\rangle_x$, $S_x^-|-\rangle_x = 0$

S_x^+: $S_x^+ = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad |-\rangle_x = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\frac{1}{\sqrt{2}}$

$S_x^+|+\rangle_x = 0 \Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\cancel{\frac{1}{\sqrt{2}}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad |+\rangle_x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}}$

$\Rightarrow a+b=0,\ c+d=0 \Rightarrow a=-b,\ c=-d$

$S_x^+|-\rangle_x = \hbar|+\rangle_x \Rightarrow \begin{pmatrix} a & -a \\ c & -c \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\cancel{\frac{1}{\sqrt{2}}} = \hbar\begin{pmatrix} 1 \\ 1 \end{pmatrix}\cancel{\frac{1}{\sqrt{2}}}$

$\Rightarrow a+a=\hbar,\ c+c=\hbar \Rightarrow a=c=\frac{\hbar}{2}$

$$\Rightarrow S_x^+ = \frac{\hbar}{2}\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$$

S_x^-: $S_x^- = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$S_x^-|-\rangle_x = 0 \Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\cancel{\frac{1}{\sqrt{2}}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} a-b=0 \\ c-d=0 \end{matrix}$

$\Rightarrow a=b,\ c=d$

$S_x^-|+\rangle_x = \hbar|-\rangle_x \Rightarrow \begin{pmatrix} a & a \\ c & c \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\cancel{\frac{1}{\sqrt{2}}} = \hbar\begin{pmatrix} 1 \\ -1 \end{pmatrix}\cancel{\frac{1}{\sqrt{2}}}$

$\Rightarrow a+a=\hbar,\ c+c=-\hbar \Rightarrow a=\frac{\hbar}{2},\ c=\frac{-\hbar}{2}$

$$\Rightarrow S_x^- = \frac{\hbar}{2}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$