**EE5** **2008**

**Homework assignment 3, revised problem 1(d-h)**

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**Problem 1: Spinors**

A beam of H-atoms is polarized in a Stern-Gerlach apparatus. (Regard the H-atom as a spin 1/2 particle). Let the $H$ atoms coming out of the polarizer be moving along the $x$ axis and assume the spin of the H-atoms points in the positive $y$ direction (this instrument corresponds to a $y-$polarizer).

a) Imagine now that the beam is directed into another Stern-Gerlach apparatus, an analyzer, that measures the $S_x$ component of the spin. Use the Pauli matrix to calculate $< S_x >$.

b) What is the state vector immediately after the measurement in a)?

c) A uniform magnetic field is turned on over a distance $L$ of the beam path after it comes out of the $y-$polarizer. The magnetic field vector $\vec{B}_0$ is pointing in the positive $z$ direction. Treating the translational degrees of freedom classically in this case, find expressions for $< S_x >$, $< S_y >$ and $< S_z >$ for the beam after it emerges out of the field (in terms of $L$, the velocity of the atoms $v$, and $\omega_0 = -\gamma B_0$). Give a physical interpretation.

In parts d) to h) use a quantum mechanical treatment of the translational degree of freedom. Assume that the magnetic field turns on and off over a short distance compared with the wavelength of the H-atom so the change is abrupt, i.e. the field is given by

$$B_z(x) = \begin{cases} 0 & \text{if} \quad x < -a \\ -B_0 & \text{if} \quad -a < x < 0 \\ 0 & \text{if} \quad x > 0 \end{cases}$$

Since the field has no $z$ and $y$ components only the translation in $x$-direction matters and the problem becomes one-dimensional.

Thus, the potential energy of the H-atom is

$$W(x) = -\vec{M} \cdot \vec{B}_0 = \omega_0 S_z \quad if \quad -a < x < 0$$

$$= 0 \qquad else.$$

where $\omega_0 = -\gamma B_0 > 0$. (Note that $\gamma < 0$).

d) Write down the effective one-dimensional Hamiltonian for H-atoms with spin parallel to the $z$ axis and also for H-atoms with spin antiparallel to the $z$ axis.

e) Write down an expression for the wave function corresponding to stationary states, $\phi_\pm(x)$, of atoms with spin either parallel or antiparallel to the z-axis. Write the form of the wave function for atoms that have gone passed the magnetic field ($x \to \infty$) as:

$$\phi_\pm(x) \;=\; B_\pm \, e^{ikx}$$

f) Give an expression in terms of $\phi_\pm(x)$ for the two component spinor $[\Psi](\vec{r})$ representing the state of a hydrogen atom in the beam.

g) The transmitted beam is directed into an analyzer that measures the spin component along a selected axis, for example $x$, $y$ or $z$. Give an expression for $< S_x >$, $< S_y >$ and $< S_z >$ in terms of $B_+$ and $B_-$. Evaluate $< S_z >$.

h) When the magnetic field is small, the value of $< S_x >$ in the transmitted beam is small except that maxima in $< S_x >$ occur at certain values for the energy of the atoms. Explain the effect.

(For extra bonus, do the calculation of $< S_x >$ vs energy!).

**Problem 2: Raising and lowering operators for spin projection**

Find the 2x2 matrices corresponding to raising and lowering operators of the $x-$projection of the spin of an electron, using the $|+>_z$ and $|->_z$ states as basis (i.e. spin-up and spin-down states along the $z-$axis).