Hlutapróf 2, 18. október, 8:20-9:50

Leyfð hjálpargögn: reiknivél og ein A4 blaðsíða sem hver nemandi hefur skrifað sjálfur.

Problem 1:

Consider a harmonic oscillator with a potential energy function $V(x) = \frac{1}{2}kx^2$ and mass μ. The state of the oscillator has been prepared in such a way that its wave function is

$$\Psi(x) \;=\; A(1-x)e^{-\alpha x^2/2}$$

where $\alpha = \sqrt{k\mu}/\hbar$.

(a) Give an expression for A so that the wave function $\Psi(x)$ is normalized.

(b) Write the wave function, $\Psi(x)$, as a linear combination of the eigenfunctions, $\Psi_n(x)$, where $n = 0, 1, \ldots$

(c) Find the probability that a measurement of the energy gives the value $\frac{1}{2}\hbar\sqrt{\frac{k}{\mu}}$.

(d) Find the probability that a measurement of the energy gives the value $\frac{3}{2}\hbar\sqrt{\frac{k}{\mu}}$.

(e) Find the probability that a measurement of the energy gives the value $\frac{5}{2}\hbar\sqrt{\frac{k}{\mu}}$.

(f) Find the expectation value of the energy.

(g) Assume now that the oscillator represents a polar diatomic molecule. Which vibrational excitations could an electromagnetic wave induce in a molecule that is initially in state $\Psi(x)$ (give quantum numbers before and after the transition(s))?

(h) The interaction between the two atoms in the molecule can be described quite accurately with a Morse potential which can be written as

$$V(r) \;=\; D\left(e^{-2\beta(r-r_0)} - 2e^{-\beta(r-r_0)}\right)$$

corresponding to a bond energy of D and bond length of r_0. Given an expression for the frequency of electromagnetic radiation that can excite the vibration of the molecule.