Hlutapróf 1, 8. október, 10:00-11:30

Leyfð hjálpargögn: reiknivél og ein A4 blaðsíða sem hver nemandi hefur skrifað sjálfur.

Problem 1: (40 pts)

Consider a particle in a box where the potential is zero between $x = 0$ and $x = a$, but infinite elsewhere. The system is prepared in such a way that the wavefunction is

$$\psi(x) = N(\sin(2\pi x/a) + 2\sin(4\pi x/a)).$$

(a) Find the value of the normalization constant, N.

(b) What is the probability that a measurement of the total energy of the system will give the ground state energy? Explain your answer.

(c) What is the expectation value of the total energy in the state $\psi(x)$?

(d) Assume that a measurement of the total energy is made and the value found is the energy of the first excited state. What is the wavefunction describing the system after the measurement?

Problem 2: (60 pts)

A hydrogen atom adsorbed on the surface of a metal crystal can be considered as a quantum particle moving in a periodic potential (the metal atoms are considered to be stationary because they are so much heavier than the hydrogen atom). The potential energy of the hydrogen atom can be approximated by the function

$$V(x) = V_s \; e^{-cos(2\pi x/b)}.$$

(a) Sketch (qualitatively) the potential energy function. Label the axes, mark the values $x = b$ and $E_{pot} = V_s$, and indicate on the x-axis the locations where the H-atom would sit if it had no kinetic energy and had minimal potential energy.

(b) Derive an expression for the force acting on the hydrogen atom at an arbitrary position x.

(c) Expand $V(x)$ in a Taylor series about one of the minima up to second order in x and give an expression for the force constant, k, of the harmonic oscillator approximation to $V(x)$ in terms of V_s, b and other constants.

(d) What is the value of the zero-point energy of the hydrogen atom within the harmonic approximation described in part (c)?

(e) What would the energy of a photon need to be in order to excite the hydrogen atom from the ground state to the first excited state (assuming the harmonic approximation is valid)?

(f) Given that the hydrogen atom only has three bound states in each of the wells in the potential $V(x)$, sketch (qualitatively) the photo-absorption spectrum for the hydrogen atom assuming that all levels are roughly equally populated before the photoexcitation?